

Skewed Distributions, Unipolar Traits,  
Excess Zeros, and IRT Modeling of  
Health Constructs

Steven P. Reise

University of California, Los Angeles

## Assumptions of IRT

IRT models are valuable psychometric tools when the model-derived latent variable scale ( $\theta$ ) accurately reflects individual differences on the trait the researcher is trying to measure and the estimated item parameters faithfully reflect the relation between trait levels and the probability of category response. This assumes:

## Assumptions of IRT

- 1) The latent trait exists and is causative (not emergent)
- 2) Item response data are locally independent/monotonic

## Assumptions of IRT

- 3) The calibration sample is homogeneous/representative.
- 4) The parametric form of the model is correct and errors are logistic (in logistic models) or normal (in normal-ogive models).
- 5) The item response data are unidimensional (one common source) and item parameters reflect that common source.

## Assumptions of IRT

6) The latent trait is continuous, bipolar, and the shape of the latent variable is properly specified during estimation. In other words, it is assumed that the latent trait is a continuous “bipolar trait” that has substantively meaningful variation across the range of the latent variable (Lucke. 2015, p. 273).

## Assumptions of IRT

The validity of the conclusions drawn from any IRT model application is threatened to the degree that any of these assumptions are violated.

## Misspecifications Caused by Non-Normality:

Woods and Thissen (2006, p. 283) nicely summarize the consequences of non-normality:

“There is fairly consistent evidence that, when normality of  $g(\theta)$  is assumed, MML estimates of more extreme item parameters (e.g., thresholds around  $\pm 2$ ) are nontrivially biased when the true population distribution is platykurtic or skewed, and if  $g(\theta)$  is skewed, the bias increases as the skewness increases.”

## Possible Causes of Non-Normality:

- Latent trait is continuous but non-normal
- Latent trait is unipolar
- Excess zeros in the data
- Heterogeneous Errors

\* These are not mutually exclusive



## Non-Normally Distributed Latent Variable in the Population.

- 1) A Ramsay Curve IRT Model (RC-IRT)
- 2) A Heteroskedastic-Skew Model (HS)

## Unipolar Constructs, Quasi-Traits, and Excess Zeros

- 3) A zero-inflated mixture Model (ZIM)
- 4) A Log-Logistic Model (LL)

The first two models were developed to address non-normal latent trait distributions and violations of homogeneity of error variance, respectively.

The second two models were developed to address modeling problems associated with “quasi-continuous” or “unipolar” constructs, which apply only to a subset of the population, or are meaningful at one end of the continuum only.

# Quasi-Traits

Almost 30 years ago Reise and Waller (1990, p. 57) stated that some “personality traits may have an inherently quasi-categorical rather than a full range continuum structure.”

Observing that clinical assessment instruments have highly peaked information functions in the high (pathological) trait range and a notable lack of items that provide discriminations among individuals in low trait ranges, Reise and Waller (2007, p. 31) stated, “we believe that the peaked information function for many clinical scales reflects the quasi-trait status of many psychopathology constructs.

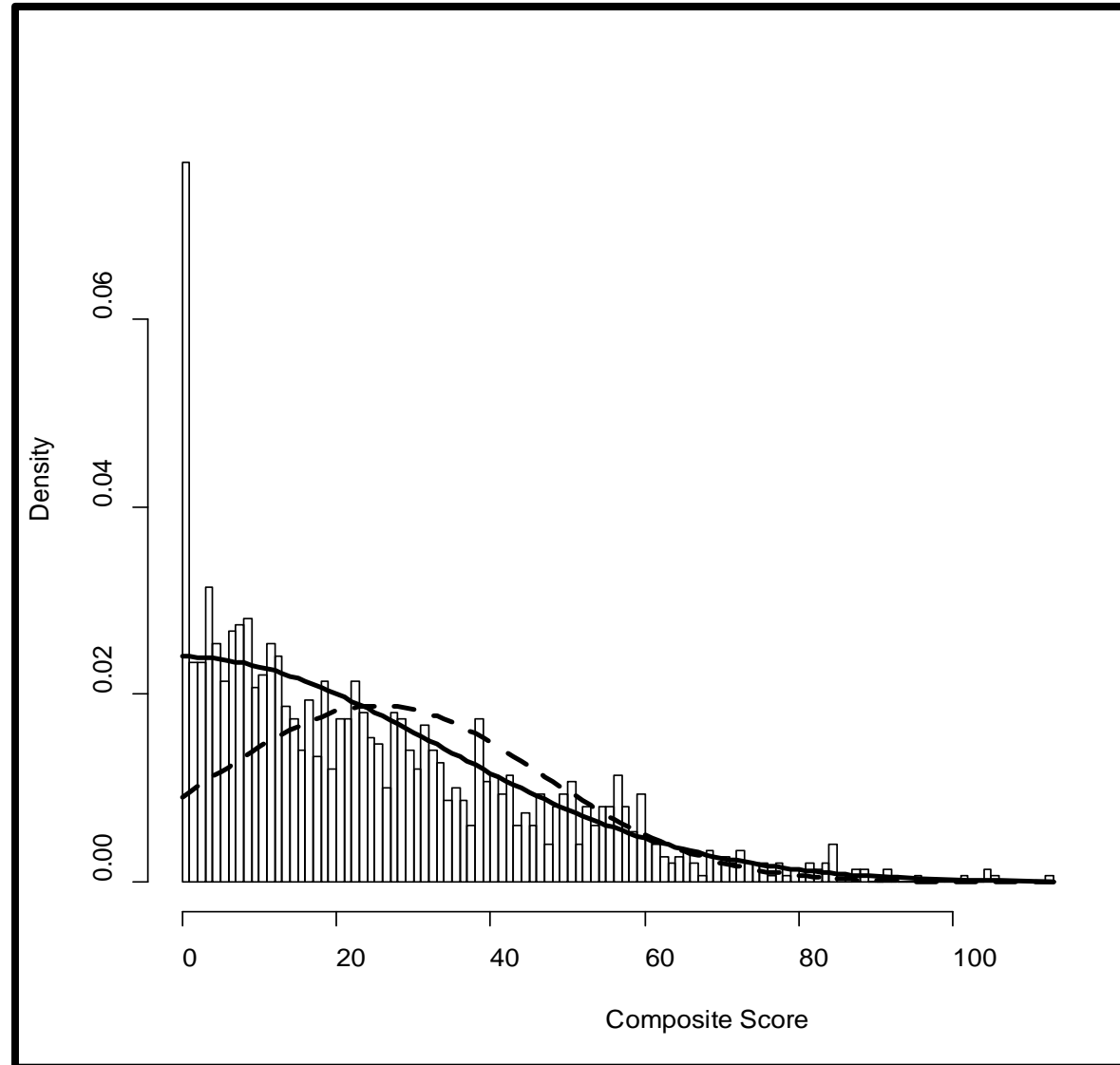
By the term ‘quasi-trait,’ we mean that the trait is unipolar (relevant only in one direction) and that variation at the low end of the scale is less informative in both a substantive as well as a psychometric sense.”

## Example Data and Model

The calibration sample consisted of 1498 non-clinical adults who responded to 29 items administered in the development of the PROMIS Anger measure.

As seen in the following figure, the distribution of summed scores appears to be more of a truncated-normal or half-normal distribution with zero-inflation; best fitting skewed normal (solid-line) and normal distributions (long-dashed line) are superimposed for illustrative purposes.

Histogram of Composite Scores for Anger Scale with Best Fitting Skewed Normal (solid line) and Normal Distribution (dashed line)



## Item Content for the PROMIS Anger Item Pool.

| Item # | PROMIS name | Content  |
|--------|-------------|--|
| 1      | edang01     | When I was frustrated, I let it show                               |
| 2      | edang03     | I was irritated more than people knew                              |
| 3      | edang04     | I felt envious of others   |
| 4      | edang05     | I disagreed with people  |
| 5      | edang06*    | I made myself angry about something just by thinking about it      |
| 6      | edang07*    | I tried to get even when I was angry with someone                  |
| 7      | edang09     | I felt angry   |
| 8      | edang10     | When I was mad at someone, I gave them the silent treatment        |
| 9      | edang11     | I felt like breaking things  |
| 10     | edang15     | I felt like I was ready to explode                                 |
| 11     | edang16     | When I was angry, I sulked   |
| 12     | edang17     | I felt resentful when I didn't get my way                          |
| 13     | edang18*    | I felt guilty about my anger                                       |
| 14     | edang21     | I felt bitter about things   |
| 15     | edang22     | I felt that people were trying to anger me                         |
| 16     | edang25*    | I stayed angry for hours   |
| 17     | edang26     | I held grudges towards others                                      |
| 18     | edang28 *   | I felt angrier than I thought I should                             |
| 19     | edang30     | I was grouchy  |
| 20     | edang31     | I was stubborn with others   |
| 21     | edang35     | I felt annoyed   |
| 22     | edang37     | I had a bad temper   |
| 23     | edang42     | I had trouble controlling my temper                                |
| 24     | edang45*    | I was angry when I was delayed                                     |
| 25     | edang47*    | Even after I expressed my anger, I had trouble forgetting about it |
| 26     | edang48     | I felt like I needed help for my anger                             |
| 27     | edang54*    | I was angry when something blocked my plans                        |
| 28     | edang55     | I felt like yelling at someone                                     |
| 29     | edang56     | Just being around people irritated me                              |

0=Never,  
 1=Rarely,  
 2=Sometimes,  
 3=Often,  
 4=Always

# Graded Response Model

# Item Slope, Location, and Intercept Parameter Estimates Under the Graded Response Model, Assuming a Normally Distributed Latent Trait.

|                   | Slope    |           | Location  |           |           | Intercept  |            |            |            |
|-------------------|----------|-----------|-----------|-----------|-----------|------------|------------|------------|------------|
|                   | $\alpha$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ |
| Item 1            | 1.71     | -0.77     | 0.40      | 1.72      | 2.98      | 1.31       | -0.69      | -2.94      | -5.09      |
| Item 2            | 2.25     | -0.40     | 0.46      | 1.52      | 2.48      | 0.90       | -1.03      | -3.42      | -5.58      |
| Item 3            | 1.80     | 0.26      | 1.18      | 2.21      | 3.26      | -0.47      | -2.12      | -3.98      | -5.86      |
| Item 4            | 1.57     | -1.21     | 0.04      | 1.96      | 3.60      | 1.90       | -0.06      | -3.08      | -5.65      |
| Item 5            | 2.65     | -0.13     | 0.70      | 1.69      | 2.55      | 0.34       | -1.86      | -4.48      | -6.75      |
| Item 6            | 2.92     | 0.47      | 1.16      | 1.81      | 2.70      | -1.38      | -3.39      | -5.28      | -7.87      |
| Item 7            | 2.85     | -0.66     | 0.42      | 1.59      | 2.75      | 1.87       | -1.20      | -4.53      | -7.86      |
| Item 8            | 1.97     | -0.17     | 0.69      | 1.68      | 2.77      | 0.34       | -1.36      | -3.32      | -5.46      |
| Item 9            | 3.14     | 0.65      | 1.13      | 1.77      | 2.65      | -2.03      | -3.55      | -5.56      | -8.32      |
| Item 10           | 3.35     | 0.18      | 0.89      | 1.62      | 2.52      | -0.61      | -2.97      | -5.42      | -8.46      |
| Item 11           | 2.76     | -0.12     | 0.71      | 1.67      | 2.53      | 0.33       | -1.95      | -4.61      | -6.98      |
| Item 12           | 2.60     | 0.08      | 0.94      | 1.85      | 2.79      | -0.20      | -2.45      | -4.81      | -7.25      |
| Item 13           | 2.25     | 0.01      | 0.76      | 1.76      | 2.52      | -0.02      | -1.72      | -3.96      | -5.67      |
| Item 14           | 2.75     | -0.05     | 0.75      | 1.63      | 2.84      | 0.14       | -2.06      | -4.48      | -7.80      |
| Item 15           | 2.62     | 0.32      | 0.97      | 1.97      | 2.94      | -0.83      | -2.54      | -5.16      | -7.69      |
| Item 16           | 3.30     | 0.21      | 0.98      | 1.75      | 2.69      | -0.70      | -3.23      | -5.78      | -8.88      |
| Item 17           | 2.62     | 0.13      | 0.90      | 1.82      | 2.55      | -0.33      | -2.34      | -4.76      | -6.67      |
| Item 18           | 3.23     | 0.01      | 0.76      | 1.64      | 2.42      | -0.03      | -2.47      | -5.30      | -7.83      |
| Item 19           | 2.49     | -0.76     | 0.39      | 1.62      | 2.75      | 1.88       | -0.98      | -4.04      | -6.86      |
| Item 20           | 2.20     | -0.28     | 0.76      | 1.90      | 3.11      | 0.62       | -1.67      | -4.17      | -6.82      |
| Item 21           | 2.19     | -1.05     | 0.20      | 1.60      | 2.96      | 2.30       | -0.45      | -3.51      | -6.47      |
| Item 22           | 3.21     | -0.08     | 0.78      | 1.68      | 2.62      | 0.25       | -2.51      | -5.38      | -8.42      |
| Item 23           | 3.27     | 0.21      | 1.01      | 1.84      | 2.54      | -0.68      | -3.31      | -6.03      | -8.31      |
| Item 24           | 2.38     | -0.24     | 0.64      | 1.69      | 2.72      | 0.57       | -1.52      | -4.01      | -6.45      |
| Item 25           | 3.00     | -0.07     | 0.72      | 1.62      | 2.47      | 0.21       | -2.17      | -4.87      | -7.41      |
| Item 26           | 3.33     | 0.59      | 1.11      | 1.84      | 2.54      | -1.98      | -3.69      | -6.12      | -8.46      |
| Item 27           | 2.68     | -0.16     | 0.73      | 1.71      | 2.61      | 0.43       | -1.96      | -4.57      | -7.00      |
| Item 28           | 2.79     | -0.17     | 0.65      | 1.65      | 2.53      | 0.47       | -1.82      | -4.60      | -7.05      |
| Item 29           | 2.42     | 0.69      | 1.34      | 2.06      | 2.79      | -1.66      | -3.23      | -4.97      | -6.75      |
| Mean              | 2.63     | -0.09     | 0.76      | 1.75      | 2.73      | 0.10       | -2.08      | -4.59      | -7.09      |
| SD                | 0.50     | 0.47      | 0.30      | 0.15      | 0.26      | 1.12       | 0.95       | 0.86       | 1.04       |
| Mean <sup>p</sup> | 2.17     | -0.19     | 0.81      | 2.04      | 3.15      |            |            |            |            |
| SD <sup>p</sup>   | 0.43     | 0.65      | 0.55      | 0.43      | 0.47      |            |            |            |            |

Above the mean!

Average is -0.09

Graded Response Model

Note.  $\alpha$  = slope,  $\beta$  = location,  $\gamma$  = intercept, <sup>p</sup> indicates Pilkonis et al. (2011) results.

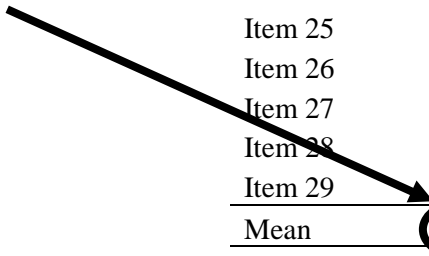


# Ramsay Curve IRT

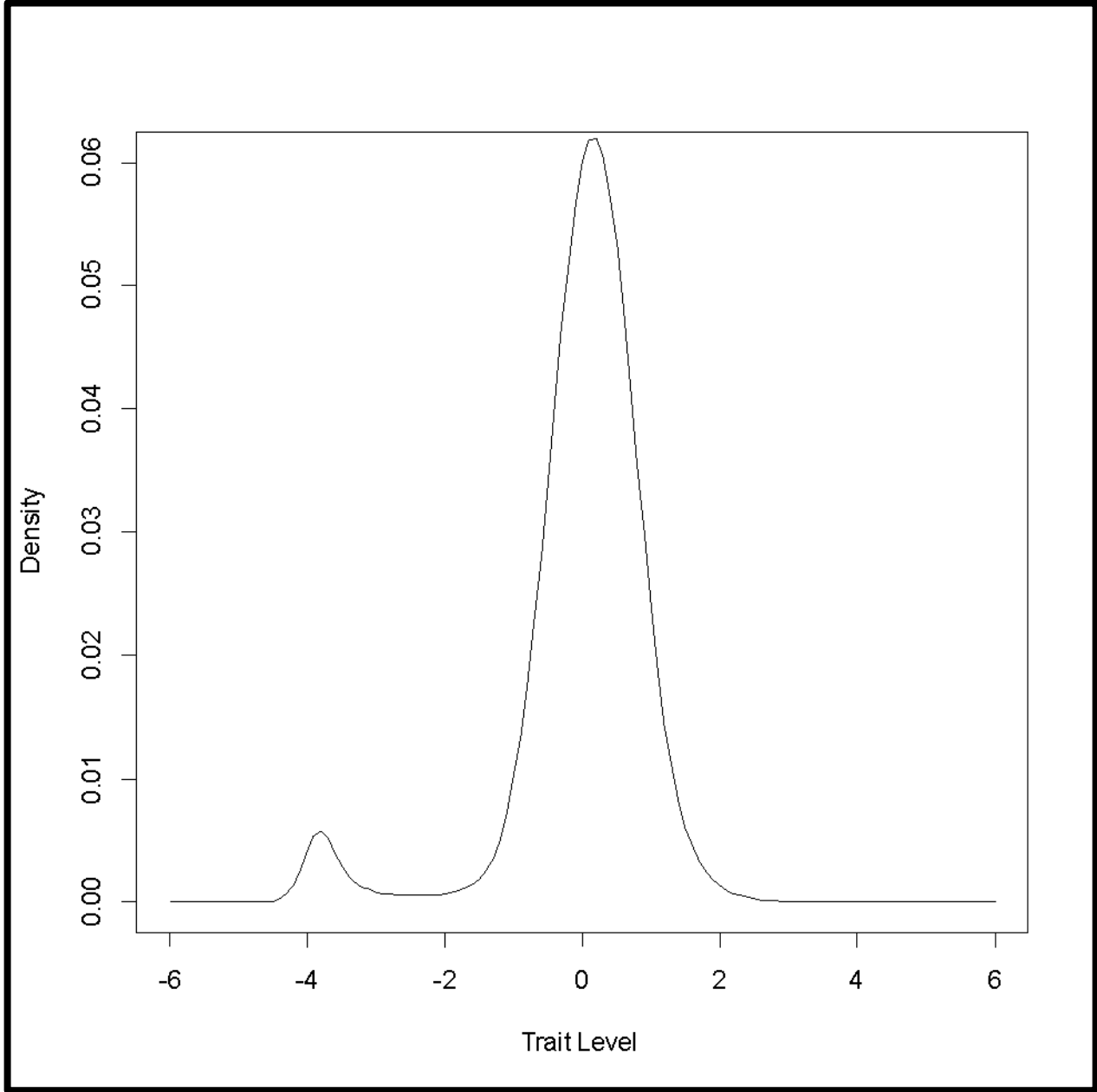
Ramsay Curve IRT GRM  
Parameter Estimates Based on  
2 degrees and 6 knots.

|         | $\alpha$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ |
|---------|----------|-----------|-----------|-----------|-----------|
| Item 1  | 2.35     | -0.47     | 0.38      | 1.33      | 2.23      |
| Item 2  | 3.19     | -0.19     | 0.42      | 1.16      | 1.84      |
| Item 3  | 2.56     | 0.28      | 0.92      | 1.65      | 2.38      |
| Item 4  | 2.09     | -0.84     | 0.11      | 1.53      | 2.74      |
| Item 5  | 3.79     | 0.01      | 0.59      | 1.28      | 1.88      |
| Item 6  | 4.19     | 0.43      | 0.91      | 1.36      | 1.98      |
| Item 7  | 4.05     | -0.36     | 0.39      | 1.21      | 2.03      |
| Item 8  | 2.79     | -0.02     | 0.58      | 1.28      | 2.04      |
| Item 9  | 4.50     | 0.55      | 0.89      | 1.33      | 1.95      |
| Item 10 | 4.82     | 0.22      | 0.71      | 1.22      | 1.86      |
| Item 11 | 3.95     | 0.02      | 0.59      | 1.27      | 1.87      |
| Item 12 | 3.72     | 0.15      | 0.76      | 1.39      | 2.05      |
| Item 13 | 3.22     | 0.10      | 0.63      | 1.33      | 1.86      |
| Item 14 | 3.93     | 0.06      | 0.62      | 1.23      | 2.09      |
| Item 15 | 3.75     | 0.32      | 0.77      | 1.47      | 2.15      |
| Item 16 | 4.75     | 0.25      | 0.78      | 1.32      | 1.98      |
| Item 17 | 3.75     | 0.19      | 0.72      | 1.37      | 1.88      |
| Item 18 | 4.63     | 0.11      | 0.63      | 1.24      | 1.79      |
| Item 19 | 3.50     | -0.44     | 0.37      | 1.24      | 2.04      |
| Item 20 | 3.12     | -0.10     | 0.63      | 1.43      | 2.28      |
| Item 21 | 3.02     | -0.67     | 0.24      | 1.23      | 2.20      |
| Item 22 | 4.61     | 0.04      | 0.64      | 1.27      | 1.93      |
| Item 23 | 4.70     | 0.24      | 0.80      | 1.38      | 1.87      |
| Item 24 | 3.38     | -0.07     | 0.54      | 1.28      | 2.00      |
| Item 25 | 4.30     | 0.05      | 0.60      | 1.23      | 1.82      |
| Item 26 | 4.78     | 0.51      | 0.87      | 1.38      | 1.87      |
| Item 27 | 3.83     | -0.01     | 0.61      | 1.29      | 1.92      |
| Item 28 | 4.00     | -0.02     | 0.55      | 1.25      | 1.86      |
| Item 29 | 3.46     | 0.58      | 1.03      | 1.53      | 2.05      |
| Mean    | 3.75     | 0.03      | 0.63      | 1.33      | 2.01      |

2.63 in GRM



The Latent Density  
Estimated by RC-  
LOG.



# Heteroskedastic-Skew Model

$$y_i^* = v_i + \lambda_i \theta + \epsilon_i$$

Parameter Estimates  
from the  
Heteroskedastic-  
Skew Baseline  
Model

|         | $\lambda$ | $\tau_3$ | $\tau_4$ | $\delta_0$ | $\delta_1^*$ | $\nu$ | $\alpha$ |
|---------|-----------|----------|----------|------------|--------------|-------|----------|
| Item 1  | 0.96      | 1.71     | 2.77     | 1.11       | 0            | 0.09  | 0.86     |
| Item 2  | 1.02      | 1.58     | 2.47     | 0.65       | 0            | 0.12  | 1.57     |
| Item 3  | 0.93      | 2.17     | 3.01     | 0.91       | 0            | 0.13  | 1.02     |
| Item 4  | 0.97      | 1.92     | 3.38     | 1.18       | 0            | 0.10  | 0.82     |
| Item 5  | 0.94      | 1.62     | 2.33     | 0.41       | 0            | 0.11  | 2.29     |
| Item 6  | 0.97      | 1.83     | 2.63     | 0.39       | 0            | 0.10  | 2.49     |
| Item 7  | 1.00      | 1.57     | 2.67     | 0.39       | 0            | 0.06  | 2.56     |
| Item 8  | 1.01      | 1.70     | 2.67     | 0.84       | 0            | 0.07  | 1.2      |
| Item 9  | 1.00      | 1.75     | 2.58     | 0.34       | 0            | 0.02  | 2.94     |
| Item 10 | 0.96      | 1.60     | 2.42     | 0.28       | 0            | 0.12  | 3.48     |
| Item 11 | 0.94      | 1.61     | 2.36     | 0.39       | 0            | 0.11  | 2.41     |
| Item 12 | 0.88      | 1.71     | 2.47     | 0.39       | 0            | 0.11  | 2.26     |
| Item 13 | 1.02      | 1.80     | 2.46     | 0.69       | 0            | 0.09  | 1.48     |
| Item 14 | 0.84      | 1.45     | 2.40     | 0.32       | 0            | 0.13  | 2.62     |
| Item 15 | 0.89      | 1.78     | 2.58     | 0.39       | 0            | 0.09  | 2.28     |
| Item 16 | 1.02      | 1.78     | 2.66     | 0.31       | 0            | 0.07  | 3.29     |
| Item 17 | 1.01      | 1.83     | 2.52     | 0.50       | 0            | 0.06  | 2.02     |
| Item 18 | 1.03      | 1.70     | 2.44     | 0.34       | 0            | 0.09  | 3.03     |
| Item 19 | 1.00      | 1.66     | 2.76     | 0.55       | 0            | 0.08  | 1.82     |
| Item 20 | 1.01      | 1.96     | 3.08     | 0.71       | 0            | 0.09  | 1.42     |
| Item 21 | 0.97      | 1.58     | 2.84     | 0.63       | 0            | 0.09  | 1.54     |
| Item 22 | 1.02      | 1.71     | 2.61     | 0.33       | 0            | 0.08  | 3.09     |
| Item 23 | 0.97      | 1.78     | 2.38     | 0.29       | 0            | 0.08  | 3.34     |
| Item 24 | 0.88      | 1.52     | 2.36     | 0.45       | 0            | 0.10  | 1.96     |
| Item 25 | 0.97      | 1.58     | 2.32     | 0.36       | 0            | 0.07  | 2.69     |
| Item 26 | 0.94      | 1.78     | 2.39     | 0.27       | 0            | 0.10  | 3.48     |
| Item 27 | 0.98      | 1.67     | 2.48     | 0.45       | 0            | 0.06  | 2.18     |
| Item 28 | 0.94      | 1.54     | 2.32     | 0.38       | 0            | 0.06  | 2.47     |
| Item 29 | 0.88      | 1.93     | 2.51     | 0.46       | 0            | 0.15  | 1.91     |
| Mean    | 0.96      | 1.72     | 2.58     | 0.51       | 0            | 0.09  | 2.22     |
| SD      | 0.05      | 0.15     | 0.25     | 0.24       | 0            | 0.03  | 0.77     |

Heteroscedasticity

Error Variance

Note. \* fixed parameter.  $\lambda$  = regression weight;  $\tau$  = threshold;  $\delta_0$  is the residual,  $\delta_1$  is the heteroscedasticity parameter; and,  $\alpha$  is the IRT slope =  $\lambda/\delta_0$ .

Parameter  
Estimates from the  
Heteroskedastic-  
Skew Full Model

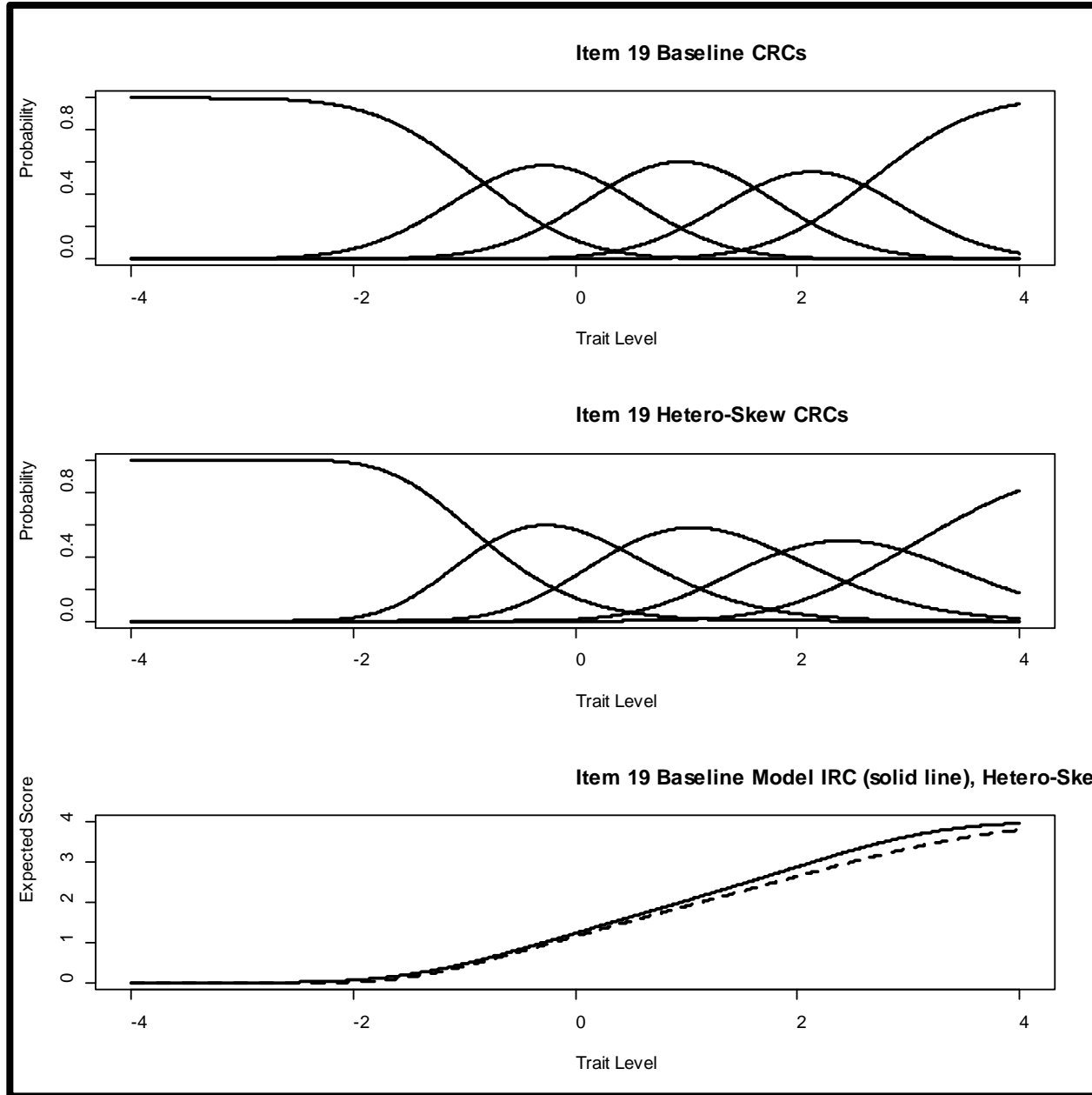
|         | $\lambda$ | $\tau_3$ | $\tau_4$ | $\delta_0$ | $\delta_1$ | $\nu$ | $\alpha$ |
|---------|-----------|----------|----------|------------|------------|-------|----------|
| Item 1  | 0.89      | 1.65     | 2.63     | 1.11       | -0.32      | -0.06 | 0.80     |
| Item 2  | 0.95      | 1.54     | 2.36     | 0.65       | -0.23      | -0.02 | 1.46     |
| Item 3  | 0.90      | 2.15     | 2.97     | 0.93       | -0.11      | -0.01 | 0.97     |
| Item 4  | 0.91      | 1.90     | 3.29     | 1.18       | -0.06      | -0.03 | 0.77     |
| Item 5  | 0.89      | 1.56     | 2.20     | 0.44       | -0.43      | -0.04 | 2.02     |
| Item 6  | 0.91      | 1.85     | 2.65     | 0.35       | 0.28       | 0.01  | 2.60     |
| Item 7  | 0.95      | 1.65     | 2.83     | 0.39       | 0.45       | -0.03 | 2.44     |
| Item 8  | 0.96      | 1.70     | 2.67     | 0.83       | 0.03       | -0.05 | 1.16     |
| Item 9  | 0.95      | 1.75     | 2.57     | 0.34       | 0.09       | -0.10 | 2.79     |
| Item 10 | 0.93      | 1.56     | 2.27     | 0.32       | -0.55      | -0.05 | 2.91     |
| Item 11 | 0.89      | 1.57     | 2.24     | 0.42       | -0.33      | -0.04 | 2.12     |
| Item 12 | 0.84      | 1.71     | 2.46     | 0.38       | 0.08       | 0.00  | 2.21     |
| Item 13 | 0.97      | 1.79     | 2.43     | 0.70       | -0.12      | -0.05 | 1.39     |
| Item 14 | 0.80      | 1.42     | 2.27     | 0.34       | -0.30      | 0.00  | 2.35     |
| Item 15 | 0.85      | 1.78     | 2.54     | 0.40       | -0.06      | -0.03 | 2.12     |
| Item 16 | 0.98      | 1.76     | 2.58     | 0.33       | -0.18      | -0.07 | 2.97     |
| Item 17 | 0.97      | 1.79     | 2.42     | 0.54       | -0.32      | -0.10 | 1.8      |
| Item 18 | 0.98      | 1.68     | 2.38     | 0.34       | -0.11      | -0.05 | 2.88     |
| Item 19 | 0.96      | 1.74     | 2.97     | 0.55       | 0.48       | -0.01 | 1.75     |
| Item 20 | 0.95      | 1.97     | 3.09     | 0.71       | 0.06       | -0.03 | 1.34     |
| Item 21 | 0.92      | 1.61     | 2.89     | 0.64       | 0.12       | -0.02 | 1.44     |
| Item 22 | 0.97      | 1.66     | 2.47     | 0.35       | -0.33      | -0.07 | 2.77     |
| Item 23 | 0.94      | 1.74     | 2.29     | 0.34       | -0.43      | -0.08 | 2.76     |
| Item 24 | 0.84      | 1.48     | 2.21     | 0.47       | -0.37      | -0.04 | 1.79     |
| Item 25 | 0.92      | 1.52     | 2.19     | 0.39       | -0.40      | -0.08 | 2.36     |
| Item 26 | 0.89      | 1.79     | 2.40     | 0.25       | 0.21       | 0.00  | 3.56     |
| Item 27 | 0.92      | 1.60     | 2.30     | 0.48       | -0.42      | -0.10 | 1.92     |
| Item 28 | 0.90      | 1.47     | 2.12     | 0.41       | -0.54      | -0.10 | 2.20     |
| Item 29 | 0.86      | 1.92     | 2.49     | 0.50       | -0.14      | 0.01  | 1.72     |
| Mean    | 0.92      | 1.70     | 2.52     | 0.52       | -0.14      | -0.04 | 2.05     |
| SD      | 0.05      | 0.16     | 0.30     | 0.24       | 0.28       | 0.03  | 0.71     |

Note. \* fixed parameter.  $\lambda$  = regression weight;  $\tau$  = threshold;  $\delta_0$  is the residual,  $\delta_1$  is the heteroscedasticity parameter; and,  $\alpha$  is the IRT slope =  $\lambda/\delta_0$ .

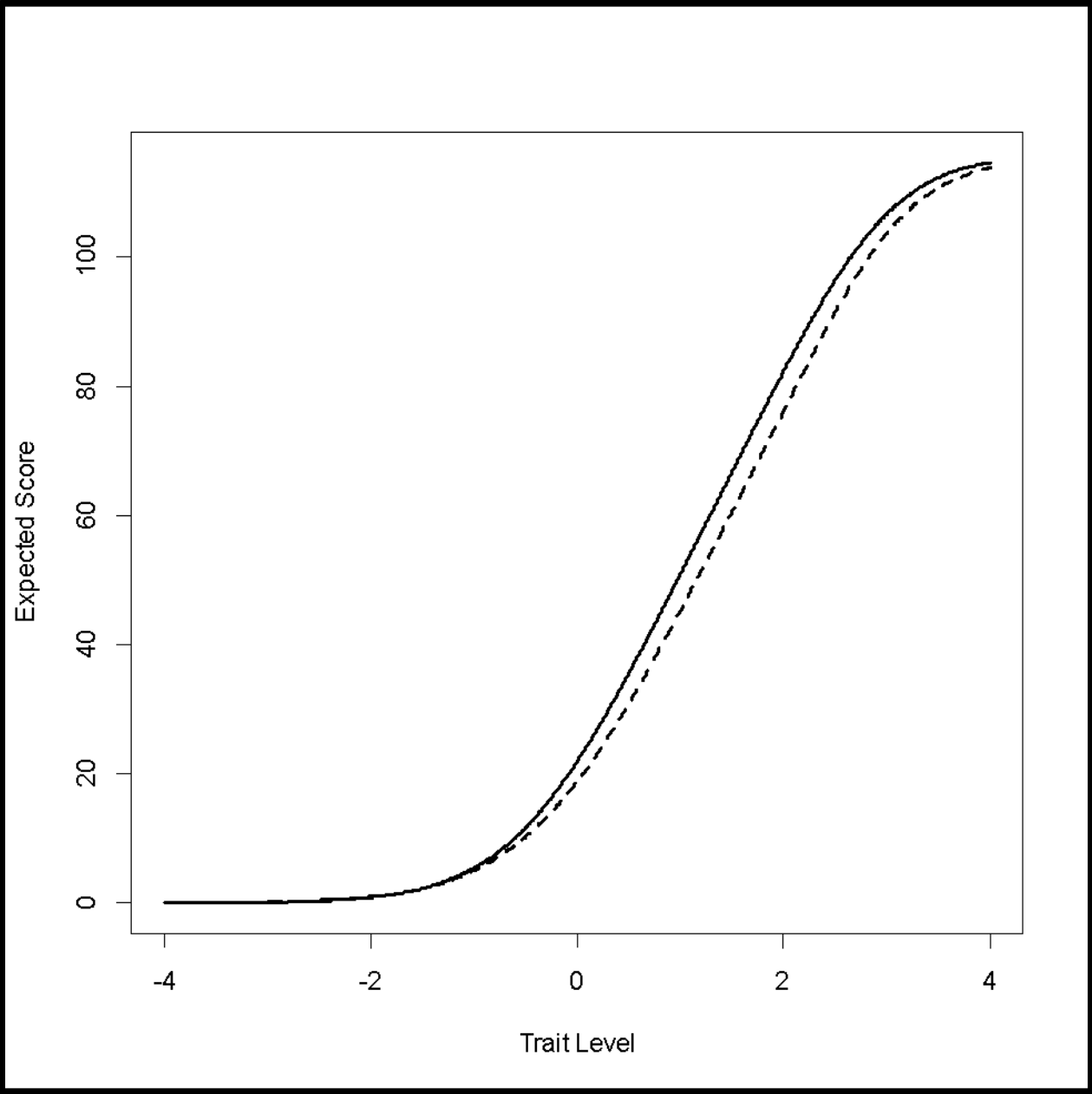
Skew Parameter  
= 0.28

The chi-square difference between the baseline and full model was 122.93 on 30 df, which is significant at  $p \leq .01$ .

CRCs and IRCs for  
Item 19 under  
Baseline Model and  
HS Model.



Test Response  
Curves for Baseline  
(solid line) and HS  
(dashed line)  
Models.





# Zero Inflated Mixture Model

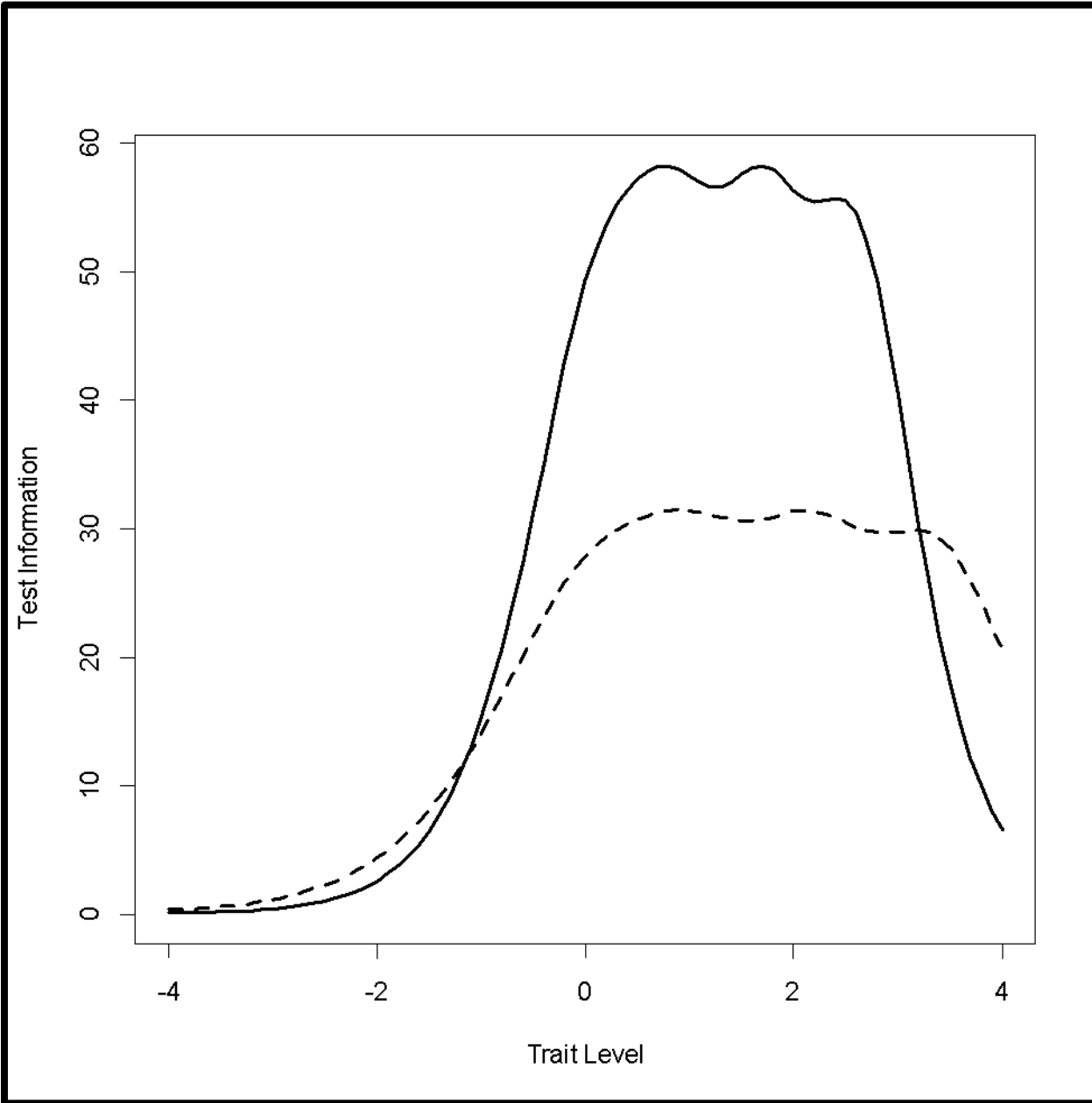
Item Slope, Location, and Intercept Parameter Estimates Under the Mixture Model, Assuming a Normally Distributed Latent Trait.

|         | Slope    |           | Location  |           |           | Intercept  |            |            |            |
|---------|----------|-----------|-----------|-----------|-----------|------------|------------|------------|------------|
|         | $\alpha$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ |
| Item 1  | 1.23     | -1.21     | 0.41      | 2.22      | 3.95      | 1.49       | -0.50      | -2.73      | -4.86      |
| Item 2  | 1.66     | -0.67     | 0.49      | 1.90      | 3.19      | 1.11       | -0.81      | -3.15      | -5.30      |
| Item 3  | 1.36     | 0.21      | 1.43      | 2.81      | 4.20      | -0.29      | -1.94      | -3.82      | -5.71      |
| Item 4  | 1.08     | -1.94     | -0.11     | 2.61      | 4.94      | 2.10       | 0.12       | -2.82      | -5.34      |
| Item 5  | 1.98     | -0.30     | 0.81      | 2.12      | 3.27      | 0.59       | -1.60      | -4.20      | -6.47      |
| Item 6  | 2.18     | 0.50      | 1.41      | 2.28      | 3.47      | -1.09      | -3.07      | -4.97      | -7.56      |
| Item 7  | 2.10     | -1.00     | 0.43      | 1.99      | 3.56      | 2.10       | -0.90      | -4.18      | -7.48      |
| Item 8  | 1.44     | -0.36     | 0.80      | 2.14      | 3.61      | 0.52       | -1.15      | -3.08      | -5.20      |
| Item 9  | 2.30     | 0.74      | 1.38      | 2.24      | 3.43      | -1.70      | -3.17      | -5.15      | -7.89      |
| Item 10 | 2.44     | 0.11      | 1.05      | 2.02      | 3.26      | -0.27      | -2.56      | -4.93      | -7.95      |
| Item 11 | 2.06     | -0.29     | 0.81      | 2.09      | 3.24      | 0.60       | -1.67      | -4.31      | -6.67      |
| Item 12 | 1.94     | -0.03     | 1.13      | 2.33      | 3.59      | 0.06       | -2.19      | -4.52      | -6.96      |
| Item 13 | 1.67     | -0.12     | 0.89      | 2.22      | 3.24      | 0.20       | -1.49      | -3.71      | -5.41      |
| Item 14 | 2.05     | -0.20     | 0.87      | 2.04      | 3.67      | 0.41       | -1.78      | -4.18      | -7.52      |
| Item 15 | 1.91     | 0.30      | 1.17      | 2.52      | 3.82      | -0.57      | -2.23      | -4.81      | -7.30      |
| Item 16 | 2.43     | 0.15      | 1.17      | 2.20      | 3.47      | -0.36      | -2.84      | -5.35      | -8.43      |
| Item 17 | 1.93     | 0.04      | 1.07      | 2.30      | 3.29      | -0.08      | -2.07      | -4.44      | -6.35      |
| Item 18 | 2.36     | -0.12     | 0.89      | 2.06      | 3.12      | 0.28       | -2.10      | -4.86      | -7.36      |
| Item 19 | 1.83     | -1.15     | 0.40      | 2.04      | 3.57      | 2.10       | -0.73      | -3.73      | -6.53      |
| Item 20 | 1.63     | -0.51     | 0.89      | 2.40      | 4.04      | 0.83       | -1.45      | -3.91      | -6.59      |
| Item 21 | 1.58     | -1.59     | 0.14      | 2.03      | 3.87      | 2.51       | -0.22      | -3.21      | -6.11      |
| Item 22 | 2.38     | -0.23     | 0.91      | 2.10      | 3.38      | 0.55       | -2.17      | -5.00      | -8.04      |
| Item 23 | 2.43     | 0.15      | 1.22      | 2.32      | 3.27      | -0.36      | -2.96      | -5.64      | -7.95      |
| Item 24 | 1.77     | -0.45     | 0.72      | 2.12      | 3.50      | 0.80       | -1.27      | -3.75      | -6.20      |
| Item 25 | 2.22     | -0.22     | 0.84      | 2.03      | 3.17      | 0.49       | -1.86      | -4.51      | -7.04      |
| Item 26 | 2.42     | 0.67      | 1.36      | 2.33      | 3.28      | -1.62      | -3.29      | -5.64      | -7.94      |
| Item 27 | 1.99     | -0.35     | 0.84      | 2.14      | 3.35      | 0.70       | -1.67      | -4.26      | -6.67      |
| Item 28 | 2.06     | -0.35     | 0.74      | 2.06      | 3.25      | 0.72       | -1.52      | -4.24      | -6.70      |
| Item 29 | 1.82     | 0.78      | 1.64      | 2.60      | 3.58      | -1.42      | -2.98      | -4.73      | -6.52      |
| Mean    | 1.94     | -0.26     | 0.89      | 2.22      | 3.54      | 0.36       | -1.80      | -4.27      | -6.76      |
| SD      | 0.37     | 0.65      | 0.40      | 0.21      | 0.39      | 1.08       | 0.90       | 0.79       | 0.98       |

Lower than GRM

Note.  $\alpha$  = slope,  $\beta$  = location,  $\gamma$  = intercept.

A Comparison of Test Information for the GRM (solid line) and the Zero-Inflated Mixture Model (dashed line).



# Log-Logistic Model

$$TRC_{ij} = P(x \geq j | \theta) = \frac{\lambda_{ij} \theta^{\eta_{ij}}}{1 + \lambda_{ij} \theta^{\eta_{ij}}}$$

$$\delta_{ij} = \left( \frac{1}{\lambda_{ij}} \right)^{\frac{1}{\eta_{ij}}}$$

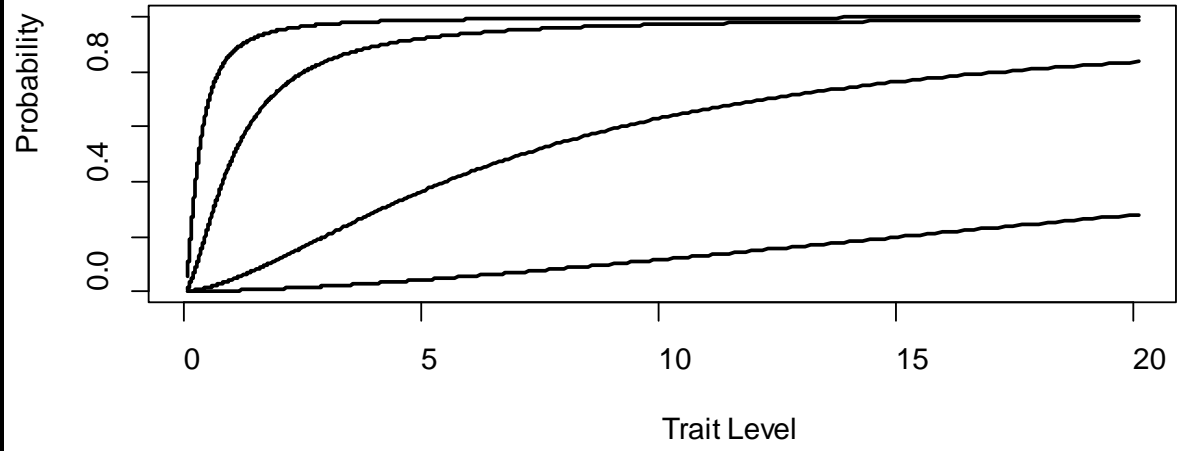
Item Slope,  
Location, and  
Intercept  
Parameter  
Estimates Under  
the  
Log-Logistic  
Model.

|         | Discrim |             | Easiness    |             |             | Location   |            |            |            |
|---------|---------|-------------|-------------|-------------|-------------|------------|------------|------------|------------|
|         | $\eta$  | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ |
| Item 1  | 1.71    | 3.706       | 0.502       | 0.053       | 0.006       | 0.463      | 1.492      | 5.585      | 19.688     |
| Item 2  | 2.25    | 2.460       | 0.357       | 0.033       | 0.004       | 0.67       | 1.584      | 4.572      | 11.941     |
| Item 3  | 1.80    | 0.625       | 0.120       | 0.019       | 0.003       | 1.297      | 3.254      | 9.116      | 26.05      |
| Item 4  | 1.57    | 6.686       | 0.942       | 0.046       | 0.004       | 0.298      | 1.041      | 7.099      | 36.598     |
| Item 5  | 2.65    | 1.405       | 0.156       | 0.011       | 0.001       | 0.878      | 2.014      | 5.419      | 12.807     |
| Item 6  | 2.92    | 0.252       | 0.034       | 0.005       | 0           | 1.6        | 3.19       | 6.11       | 14.88      |
| Item 7  | 2.85    | 6.488       | 0.301       | 0.011       | 0           | 0.517      | 1.522      | 4.904      | 15.643     |
| Item 8  | 1.97    | 1.405       | 0.257       | 0.036       | 0.004       | 0.844      | 1.994      | 5.366      | 15.959     |
| Item 9  | 3.14    | 0.131       | 0.029       | 0.004       | 0           | 1.916      | 3.096      | 5.871      | 14.154     |
| Item 10 | 3.35    | 0.543       | 0.051       | 0.004       | 0           | 1.197      | 2.435      | 5.053      | 12.429     |
| Item 11 | 2.76    | 1.391       | 0.142       | 0.010       | 0.001       | 0.887      | 2.034      | 5.312      | 12.554     |
| Item 12 | 2.60    | 0.819       | 0.086       | 0.008       | 0.001       | 1.083      | 2.56       | 6.36       | 16.281     |
| Item 13 | 2.25    | 0.980       | 0.179       | 0.019       | 0.003       | 1.01       | 2.138      | 5.812      | 12.429     |
| Item 14 | 2.75    | 1.150       | 0.127       | 0.011       | 0           | 0.951      | 2.117      | 5.104      | 17.116     |
| Item 15 | 2.62    | 0.436       | 0.079       | 0.006       | 0           | 1.377      | 2.638      | 7.171      | 18.916     |
| Item 16 | 3.30    | 0.497       | 0.040       | 0.003       | 0           | 1.234      | 2.664      | 5.755      | 14.732     |
| Item 17 | 2.62    | 0.719       | 0.096       | 0.009       | 0.001       | 1.139      | 2.46       | 6.172      | 12.807     |
| Item 18 | 3.23    | 0.97        | 0.085       | 0.005       | 0           | 1.01       | 2.138      | 5.155      | 11.246     |
| Item 19 | 2.49    | 6.554       | 0.375       | 0.018       | 0.001       | 0.468      | 1.477      | 5.053      | 15.643     |
| Item 20 | 2.20    | 1.859       | 0.188       | 0.015       | 0.001       | 0.756      | 2.138      | 6.686      | 22.421     |
| Item 21 | 2.19    | 9.974       | 0.638       | 0.030       | 0.002       | 0.35       | 1.221      | 4.953      | 19.298     |
| Item 22 | 3.21    | 1.284       | 0.081       | 0.005       | 0           | 0.923      | 2.181      | 5.366      | 13.736     |
| Item 23 | 3.27    | 0.507       | 0.037       | 0.002       | 0           | 1.234      | 2.746      | 6.297      | 12.68      |
| Item 24 | 2.38    | 1.768       | 0.219       | 0.018       | 0.002       | 0.787      | 1.896      | 5.419      | 15.18      |
| Item 25 | 3.00    | 1.234       | 0.114       | 0.008       | 0.001       | 0.932      | 2.054      | 5.053      | 11.822     |
| Item 26 | 3.33    | 0.138       | 0.025       | 0.002       | 0           | 1.804      | 3.034      | 6.297      | 12.68      |
| Item 27 | 2.68    | 1.537       | 0.141       | 0.010       | 0.001       | 0.852      | 2.075      | 5.529      | 13.599     |
| Item 28 | 2.79    | 1.600       | 0.162       | 0.010       | 0.001       | 0.844      | 1.916      | 5.207      | 12.554     |
| Item 29 | 2.42    | 0.190       | 0.040       | 0.007       | 0.001       | 1.994      | 3.819      | 7.846      | 16.281     |
| Mean    | 2.631   | 1.976       | 0.193       | 0.014       | 0.001       | 1.011      | 2.239      | 5.85       | 15.935     |
| SD      | 0.501   | 2.407       | 0.206       | 0.013       | 0.001       | 0.434      | 0.644      | 0.991      | 5.234      |

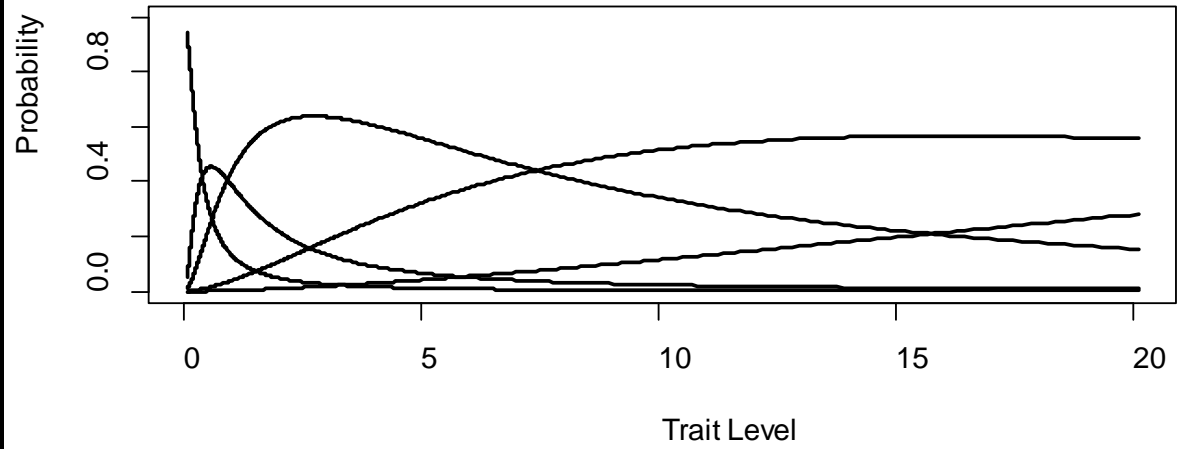
Note.  $\eta$  = slope,  $\lambda$  = easiness,  $\delta$  = location

The point on the latent  
trait continuum where the  
probability of endorsing is  
.50

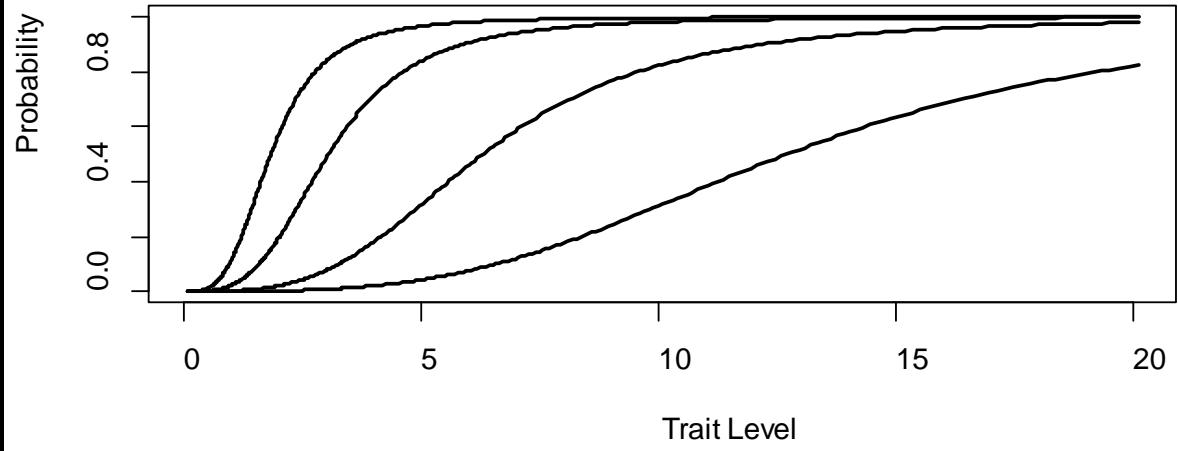
**Threshold Response Curves Item I**



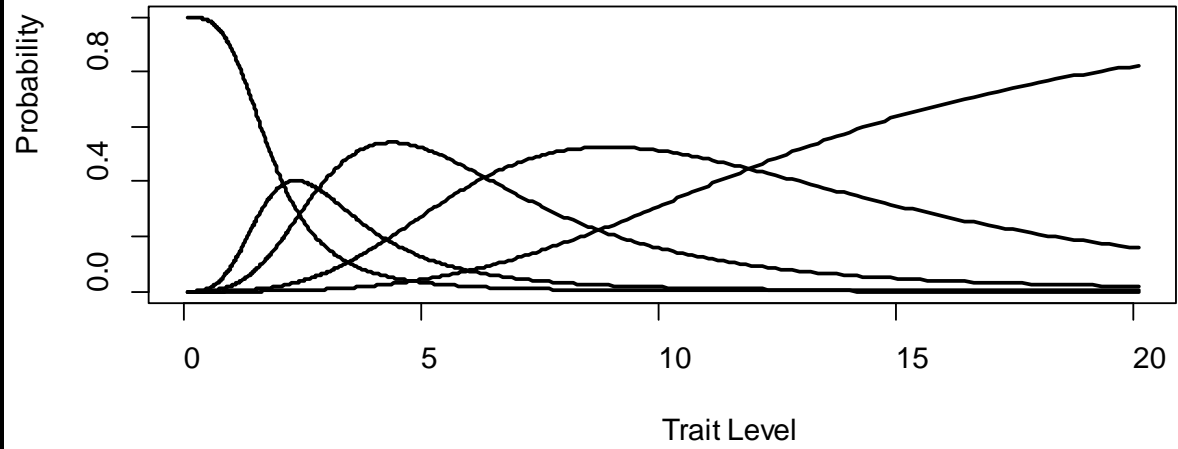
**Category Response Curves Item N**



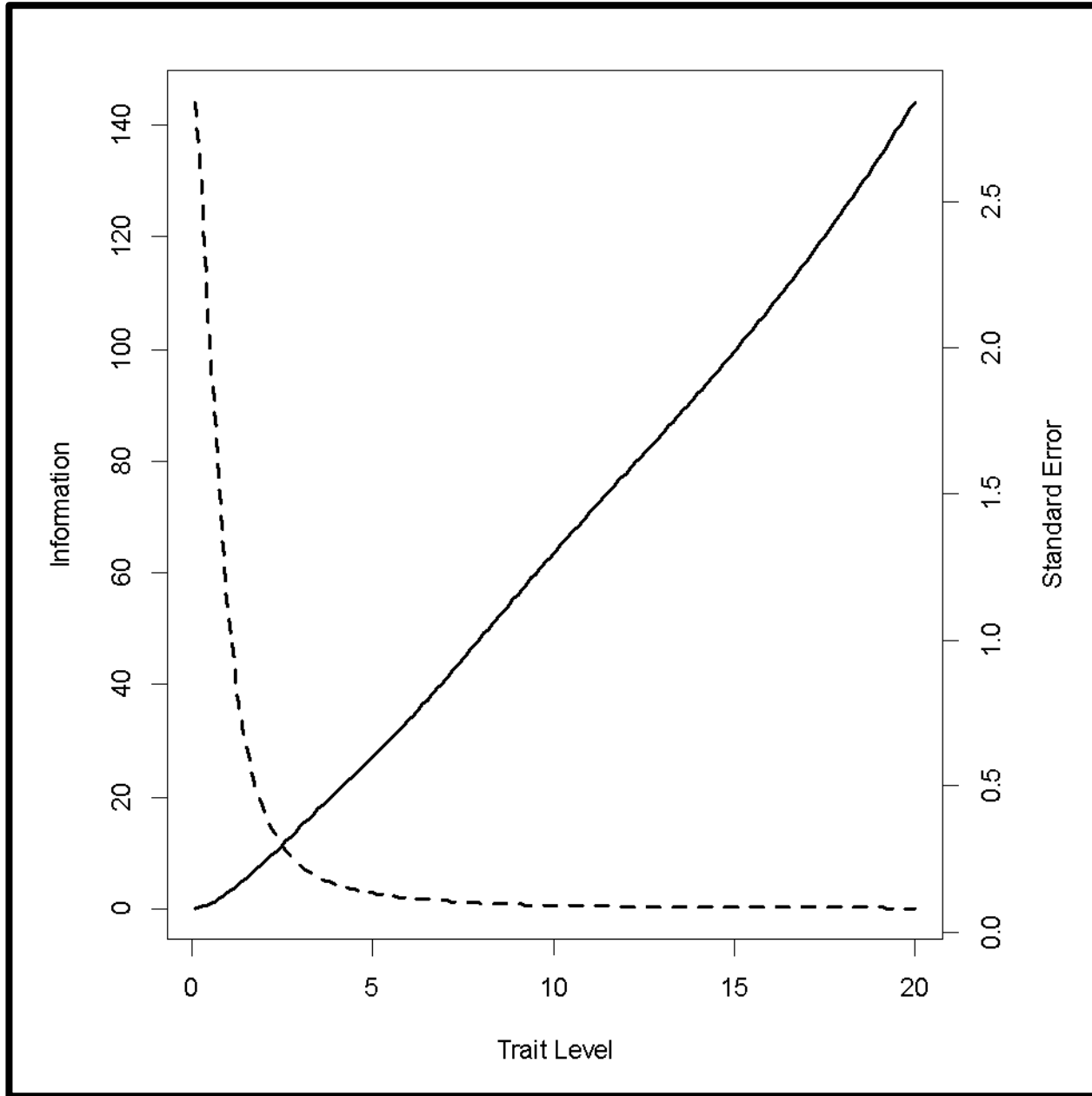
**Threshold Response Curves Item 2**



**Threshold Response Curves Item 2**



Test Information (dashed line) and Standard Errors (solid line) in the Log-Logistic Model.





## Limitations

### RC-IRT

Can only be expected to work if there is a reasonable non-normal latent distribution. If there are excess zeros, or extreme skew, then results may be highly erroneous

### HS Model

Based on a skewed-normal distribution which limits to a skewness of 1.0.

Unclear what the interpretation of heterogeneous skew is

No readily accessible parameter estimation routines

## Limitations

### Mixture-IRT

Partitions most zero responses to a degenerate class (no scores), but some zeros are considered part of traited class

Assumes a normal distribution for the latent trait in one class

### Log-Logistic

Anchors scale at zero, no z-scores (can also be a strength)

Doesn't solve excess zeros problem, parameters are still inflated.

Information function has strange characteristics